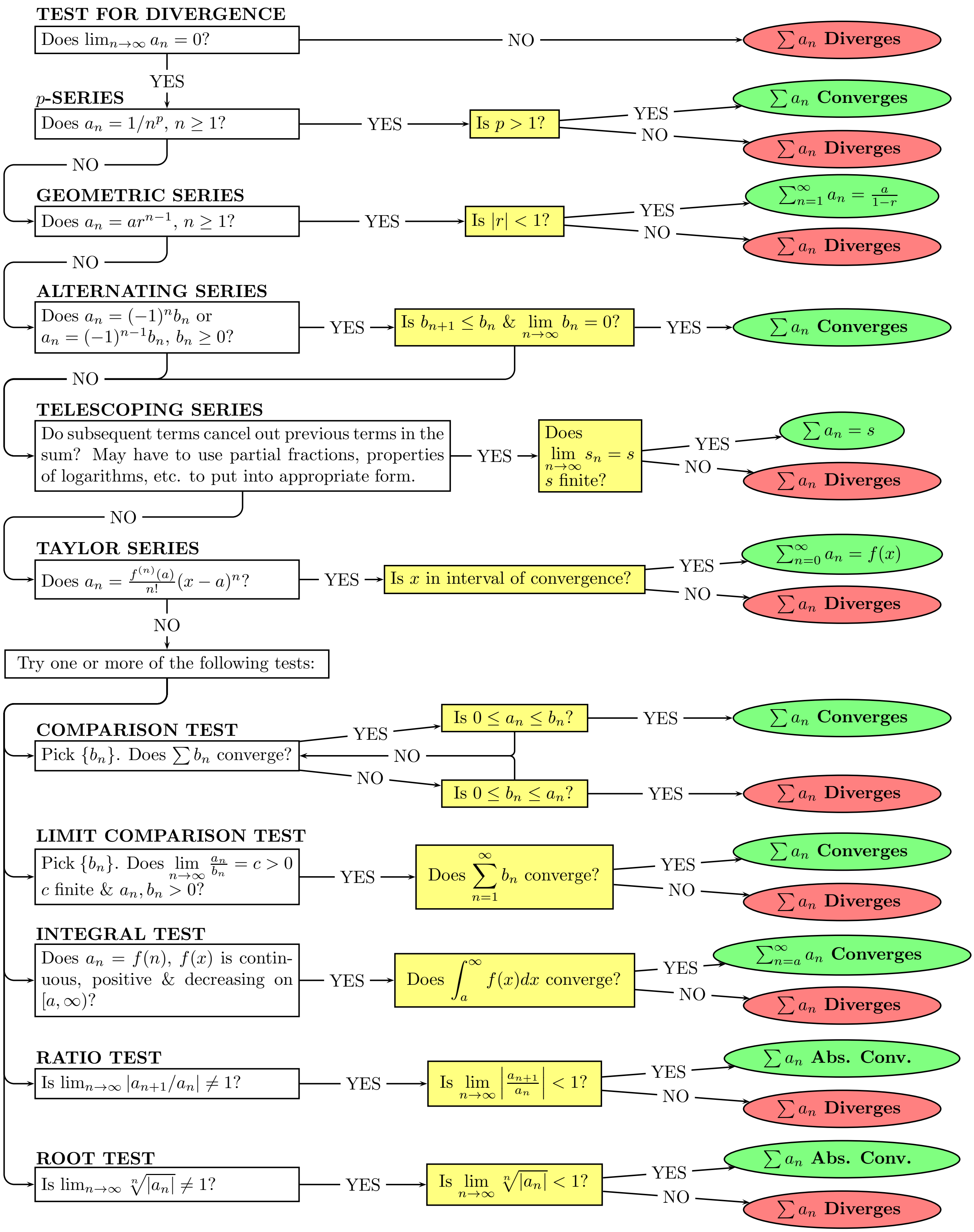
Summary of Tests for Convergence of Series

Geometric Series	The geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ with ratio r converges for $-1 < r < 1$ and diverges for $\|r\| \geq 1$.	The sum of the convergent geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ is $\frac{a}{1-r}$. The N-th partial sum is given by $S_N = \sum_{k=1}^{N} ar^{k-1} = a \cdot \frac{1-r^N}{1-r}$
p-Series	The p-series $\sum \frac{1}{k^p}$ converges if $p > 1$ and diverges if $p \leq 1$. This includes the special case of the *Harmonic series* for $p = 1$, which diverges.	
Divergence Test	If the sequence a_n does not converge to 0, then the series $\sum a_k$ diverges.	This is the first test to apply because the conclusion is simple. However, if $\lim_{n\to\infty} a_n = 0$, no conclusion can be drawn.
Integral Test	Let f be a positive, decreasing function on an interval $[c, \infty]$ and let $a_k = f(k)$ for each positive integer $k \geq c$. • If $\int_c^{\infty} f(t)\, dt$ converges, then $\sum a_k$ converges. • If $\int_c^{\infty} f(t)\, dt$ diverges, then $\sum a_k$ diverges.	Use this test when $f(x)$ is easy to integrate.
Direct Comparison Test	Let $0 \leq a_k \leq b_k$ for each positive integer k. • If $\sum b_k$ converges, then $\sum a_k$ converges. • If $\sum a_k$ diverges, then $\sum b_k$ diverges.	Use this test when you have a series with known behavior that you can compare to – this test can be difficult to apply.
Limit Comparison Test	Let a_n and b_n be sequences of positive terms. If $$\lim_{k\to\infty} \frac{a_k}{b_k} = L$$ for some positive finite number L, then the two series $\sum a_k$ and $\sum b_k$ either both converge or both diverge.	Easier to apply in general than the comparison test, but you must have a series with known behavior to compare. Useful to apply to series of rational functions.
Ratio Test	Let $a_k \neq 0$ for each k and suppose $$\lim_{k\to\infty} \frac{\|a_{k+1}\|}{\|a_k\|} = r.$$ • If $r < 1$, then the series $\sum a_k$ converges absolutely. • If $r > 1$, then the series $\sum a_k$ diverges. • If $r = 1$, then test is inconclusive.	This test is useful when a series involves factorials and powers.
Root Test	Let $a_k \geq 0$ for each k and suppose $$\lim_{k\to\infty} \sqrt[k]{a_k} = r.$$ • If $r < 1$, then the series $\sum a_k$ converges. • If $r > 1$, then the series $\sum a_k$ diverges. • If $r = 1$, then test is inconclusive.	In general, the Ratio Test can usually be used in place of the Root Test. However, the Root Test can be quick to use when a_k involves kth powers.
Alternating Series Test	If a_n is a positive, decreasing sequence so that $\lim_{n\to\infty} a_n = 0$, then the alternating series $\sum (-1)^{k+1} a_k$ converges.	This test applies only to alternating series – we assume that the terms a_n are all positive and that the sequence $\{a_n\}$ is decreasing.
Alternating Series Estimation Theorem	Let $S_n = \sum_{k=1}^{n} (-1)^{k+1} a_k$ be the nth partial sum of the alternating series $\sum_{k=1}^{\infty} (-1)^{k+1} a_k$. Assume $a_n > 0$ for each positive integer n, the sequence a_n decreases to 0 and $\lim_{n\to\infty} S_n = S$. Then it follows that $\|S - S_n\| < a_{n+1}$.	This bound can be used to determine the accuracy of the partial sum S_n as an approximation of the sum of a convergent alternating series.

Based on an excerpt from Active Calculus by Matthew Boelkins, https://activecalculus.org/single/

Series Convergence/Divergence Flow Chart



Flowchart by Ralph Freese, http://math.hawaii.edu/~ralph/