

# Properties of Trigonometric Functions

## Trigonometric Functions of Angles

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

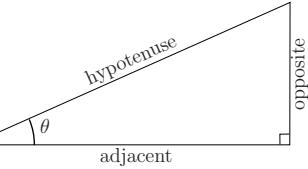
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta} = \frac{\text{adj}}{\text{opp}}$$

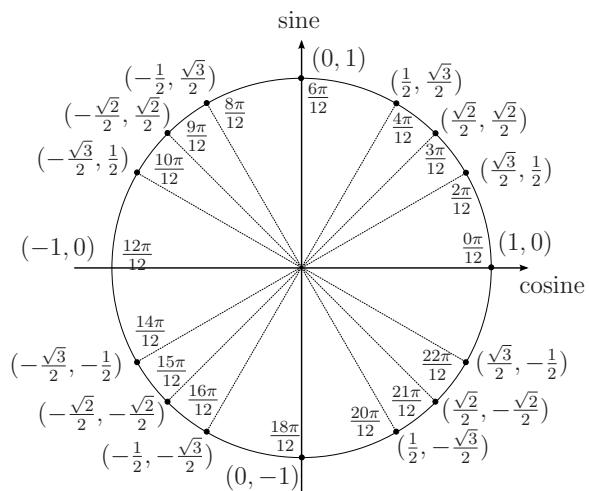


## Relationship between Degrees and Radians

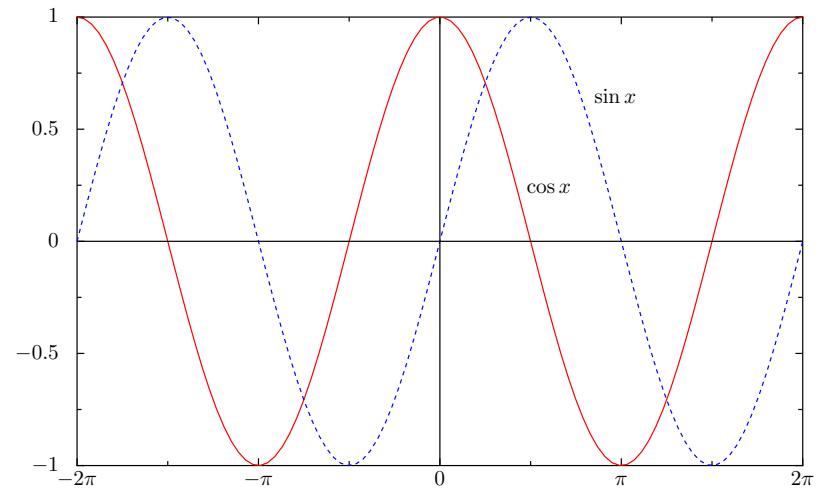
- To convert radians to degrees, multiply by  $\frac{180^\circ}{\pi}$ .
- To convert degrees to radians, multiply by  $\frac{\pi}{180^\circ}$ .

## Trigonometric Functions of Special Angles

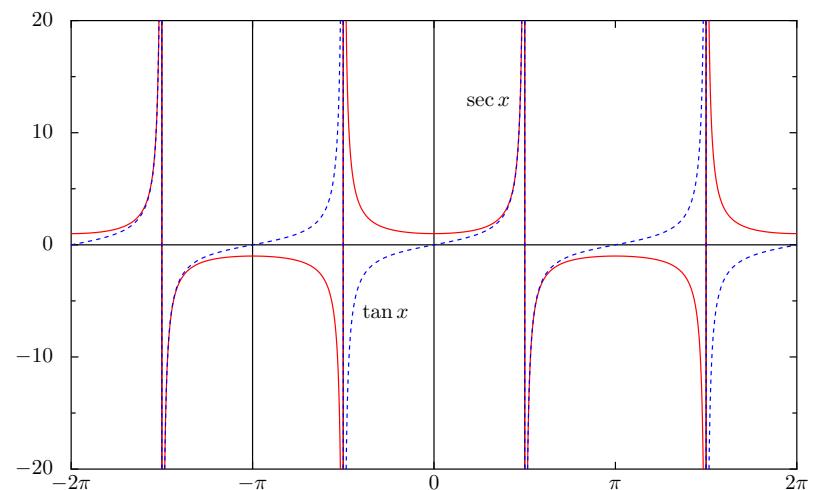
$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$180^\circ$	$270^\circ$
$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		0	



## Properties of the Basic Graphs



Domain:  $(-\infty, \infty)$ , Range:  $[-1, 1]$ , Period:  $2\pi$ , Amplitude: 1.



Domain:  $\{x \mid x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots\}$ , Range:  $(-\infty, \infty)$ ,  
Period(tan x):  $\pi$ , Period(sec x):  $2\pi$ .

## Trigonometric Identities

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### Pythagorean Identities:

$$\begin{aligned}\sin^2 x + \cos^2 x &= 1 & (\sin^2 x = 1 - \cos^2 x, \quad \cos^2 x = 1 - \sin^2 x) \\ \tan^2 x + 1 &= \sec^2 x & (\tan^2 x = \sec^2 x - 1) \\ 1 + \cot^2 x &= \csc^2 x & (\cot^2 x = \csc^2 x - 1)\end{aligned}$$

### Identities for Negatives:

$$\begin{aligned}\sin(-x) &= -\sin x & \cos(-x) &= \cos x & \tan(-x) &= -\tan x \\ \csc(-x) &= -\csc x & \sec(-x) &= \sec x & \cot(-x) &= -\cot x\end{aligned}$$

### Addition/Subtraction Formulas:

$$\begin{aligned}\cos(A+B) &= \cos A \cos B - \sin A \sin B & \cos(A-B) &= \cos A \cos B + \sin A \sin B \\ \sin(A+B) &= \sin A \cos B + \cos A \sin B & \sin(A-B) &= \sin A \cos B - \cos A \sin B \\ \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} & \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B}\end{aligned}$$

### Double-Angle Formulas:

$$\begin{aligned}\sin(2x) &= 2 \sin x \cos x \\ \cos(2x) &= \cos^2 x - \sin^2 x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1 \\ \tan(2x) &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$

### Half-Angle Formulas:

$$\begin{aligned}\sin^2 x &= \frac{1 - \cos(2x)}{2} & \sin\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 - \cos x}{2}} \\ \cos^2 x &= \frac{1 + \cos(2x)}{2} & \cos\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 + \cos x}{2}}\end{aligned}$$

## Derivatives of Trigonometric Functions

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$$\begin{aligned}\frac{d}{dx}(\sin x) &= \cos x & \frac{d}{dx}(\sin g(x)) &= \cos g(x) \cdot g'(x) \\ \frac{d}{dx}(\cos x) &= -\sin x & \frac{d}{dx}(\cos g(x)) &= -\sin g(x) \cdot g'(x) \\ \frac{d}{dx}(\tan x) &= \sec^2 x & \frac{d}{dx}(\tan g(x)) &= \sec^2 g(x) \cdot g'(x) \\ \frac{d}{dx}(\sec x) &= \sec x \tan x & \frac{d}{dx}(\sec g(x)) &= \sec g(x) \tan g(x) \cdot g'(x) \\ \frac{d}{dx}(\csc x) &= -\csc x \cot x & \frac{d}{dx}(\csc g(x)) &= -\csc g(x) \cot g(x) \cdot g'(x) \\ \frac{d}{dx}(\cot x) &= -\csc^2 x & \frac{d}{dx}(\cot g(x)) &= -\csc^2 g(x) \cdot g'(x)\end{aligned}$$

### Anti-derivatives of Trigonometric Functions

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$$\begin{aligned}\int \sin x \, dx &= -\cos x + C & \int \sec x \, dx &= \ln |\sec x + \tan x| + C \\ \int \cos x \, dx &= \sin x + C & \int \csc x \, dx &= \ln |\csc x - \cot x| + C \\ \int \tan x \, dx &= -\ln |\cos x| + C & \int \cot x \, dx &= \ln |\sin x| + C \\ \int \sec^2 x \, dx &= \tan x + C & \int \csc^2 x \, dx &= -\cot x + C \\ \int \sec x \tan x \, dx &= \sec x + C & \int \csc x \cot x \, dx &= -\csc x + C\end{aligned}$$